WELFARE THEORY: HISTORY AND MODERN RESULTS

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Contents

Summary

The purpose of this chapter is to give an overview of modern welfare theory or, at least, a significant part thereof. We start by giving the reader a historical perspective on the questions dealt with in a more formal way later on. Having done that, the remaining part of the chapter deals with issues and tools that any user of modern welfare theory must be aware of, such as the First and Second Welfare Theorems, Arrow's Impossibility Theorem, and situations were the markets themselves do not give rise to an optimal resource allocation from society's point of view. The latter is exemplified by externalities and public goods. We have also taken further steps by introducing social accounting and the associated problem of measuring welfare - a growing area of research in welfare economics - as well as introduced methods for cost benefit analysis in dynamic economies.

1. Introduction

It is reasonable to say that Adam Smith (1776) has played an important role in the development of welfare theory. The reasons are at least two. In the first place, he created the invisible hand idea that is one of the most fundamental equilibrating relations in Economic Theory; the equalization of rates of returns as enforced by a tendency of factors to move from low to high returns through the allocation of capital to individual industries by self-interested investors. The self-interest will result in an optimal allocation of capital for society. He writes: "Every individual is continually exerting himself to find out the most advantageous employment for whatever capital he can command. It is his own advantage, indeed, and not that of society, which he has in view. But the study of his own advantage naturally, or rather necessarily leads him to prefer that employment which is most advantageous to society".

He does not stop there but notes that what is true for investment is true for economic activity in general. "Every individual necessarily labours to render the annual revenue of the society as great as he can. He generally, indeed, neither intends to promote the public interest, nor knows how much he is promoting it". He concludes: "It is not from the benevolence of the butcher, the brewer, or the baker, that we expect our dinner, but from the regard of their own interest". The most famous line is probably the following: The individual is "led by an invisible hand to promote an end which was no part of his intention". The invisible hand is competition, and this idea was present already in the work of the brilliant and undervalued Irish economist Richard Cantillon. He sees the invisible hand as embodied in a central planner, guiding the economy to a social optimum. (Little is known of Cantillon's early life. He was born probably between 1680 and 1690, the second son of an Irish nobleman. His most famous work is the "Essai sur la Nature du Commerce en General". The official year of publication is 1755, but this is 21 years after his death. A possible reason for this discrepancy is found in Niehans (1990). Cantillon is perhaps most famous for his insights in monetary theory and his general equilibrium theory of the three rents. An input output system more elaborated than that of Quesnay which was sketched around 1757.).

The second reason why Adam Smith played an important role in the development of welfare theory is that, in an attempt to explain the "Water and Diamond Paradox", he came across an important distinction in value theory. At the end of the fourth chapter of the first book in Adam Smith's celebrated volume *The Wealth of Nations (1776)*, he brings up a valuation problem that is usually referred to as *The Value Paradox*. (First mentioned by John Law (1705).). He writes

"The word VALUE, it is to be observed, has two different meanings, and sometimes expresses the utility of some particular object, and sometimes the power of purchasing other goods which the possession of that object conveys. The one may be called "value in use"; the other, "value in exchange". The things which have the greatest value in use have frequently little or no value in exchange; and, on the contrary, those which have the greatest value in exchange have frequently little or no value in use. Nothing is more useful than water: but it will purchase scarce anything; scarce anything can be had in exchange for it. A diamond on the contrary, has scarce any value in use; but a very great quantity of other goods may frequently be had in exchange for it." (Smith (1776), reprinted as Peguin Classics 1986) page 131-132.).

He is unable to credibly resolve the paradox - although he uses three chapters to convince the reader that it can be resolved by the components of the natural price, i.e., essentially the notion that the long-run price is determined by the production costs. Some of the reasons behind the "failure" are not farfetched. Adam Smith was aware of supply and demand without being able to produce anything fresh about the fundamental ideas upon which these concepts rest. He was not aware of the idea to model the total utility value of consumption in terms of a utility function, and the related idea of assuming that the utility function exhibits a declining marginal utility.

Rather, it was Jules Dupuit (1844) and Heinrich Gossen (1854), who founded the modern utilitarian framework in Economics. Adam Smith's distinction between value-in-use and value-in-exchange nevertheless contain a non-trivial insight, which is fundamental for the answer to the Water and Diamond Paradox. The value in exchange is not enough to measure welfare. As Dupuit pointed out, you also need the consumer surplus which is the difference between the total surplus and the value in exchange.

The next important step in the development of welfare theory was unmistakably achieved by Leon Walras (1874). He introduced the full fledged general equilibrium system based on the fundamental principles of utility maximization and profit maximization (firms). He showed that only relative prices could be determined, since only relative prices affects the actions of firms and consumers, which means that an n-goods system has only n-1 independent equations. A point that Walras expressed by picking one good as numeraire which conveniently can be given unit price. He also showed that the budget constraint of each consumer and the objective functions facing the firms together imply that the market value of supply equals the market value of demand independent of the price vector. Most modern welfare results are in one way or another connected to the competitive general equilibrium system.

However, what was still missing after Walras had done the unifying job in his magnum opus *Element d´Economie Politique Pure*, was an idea how to rank different general equilibrium allocations. Economists had long been aware of that distributional issues matter, and that Jeremy Bentham's idea of maximizing utility of the maximum number of people typically involves one maximum too many to be feasible. It was Vilfredo Pareto, who took the distributional issue quite a bit further. He made two key contributions to existing theory. First, he realized that it was not necessary, as was implicit in 19th century Economics, that utility was cardinal, i.e., measurable in the manner which makes interpersonal utility comparisons possible. It was enough for deriving demand functions that utility was ordinal, i.e., only the individuals' rankings of different commodity bundles matter (utilities are ranked not their differences). As Pareto (1909) expressed it in the second edition of his *Manuel d'Economie Politique* (The first Italian edition appeared in 1906. The appendix of this book is best compared with Paul Samuelson, *Foundations of Economic Analysis* from (1947). For any utility function, $u(x)$, a monotone transformation is given by $F(u(x))$, with $F^{'} > 0$.): "The individual may disappear, provided he leaves us that photograph of his tastes". More formally, any monotone transformation of the utility function will result in the same demand vector. (This was not at the time, as Niehan's (1990) puts it, a revolutionary insight . It had been noted ten years later by the American Economist Irving Fisher in his thesis from 1892 published as *Mathematical Investigations in the Theory of Value and Prices* (1925).). The second and most important contribution is a partial ordering that admitted inter-personal welfare comparisons. He proposed that welfare increases if some people gain and nobody loses. Welfare declines if some people lose and nobody gains. If some gain and some lose, the welfare change is ambiguous, no verdict. This partial ordering was later called the Pareto criterion.

Clearly, it is always favorable to exhaust all mutually advantageous trades, and the resulting state is called Pareto optimal. Pareto realized that there are typically many such states starting from a given allocation of the initial resources. To illustrate Pareto optimality he could have used the concept of the contract curve that was invented 25 years earlier by Francis Ysidro Edgeworth (1881), but he used both the contract curve and a box that strangely enough today is called an Edgeworth box, perhaps since it encompasses the contract curve.

Now the time had come to produce modern welfare theory, and the man who did it was a student of Alfred Marshall and a classmate of John Maynard Keynes at Cambridge, England, namely Arthur Cecil Pigou. In Wealth and Welfare (1912) he discussed how a judicious government can increase welfare. The full fledged version of the modern welfare theory was fleshed out in *The Economics of Welfare* (1920). Apart from containing most of the relevant welfare results that follow from the Pareto criterion and Walras' general equilibrium system it also, by introducing externalities and showing how they can be handled by environmental taxes, foreshadowing modern environmental economics by almost 50 years. The welfare optimizing (improving) taxes under externalities are today called Pigouvian taxes (Pigouvian taxes were first introduced by the Dane Jens Warming (1911). He used them to solve for the first best allocation in an open access fishery model.). The driving force behind Pigou's contribution to welfare economics is his distinction between private and social cost. If they coincide the invisible hand, driven by self-interest, will tend to bring about an efficient allocation of resources (first best). In reality, with existing externalities (both positive and negative), there is room for improvement of the allocation by e.g. environmental taxes or subsidies. Another of Pigou's contributions, *A Study in Public Finance* (1947), contains fundamental insights with respect to public good provision; in particular, how the use of distortionary taxation modifies the cost benefit analysis underlying the supply of public goods. These ideas were later developed by other researchers into the concept of 'marginal cost of public funds'.

Pigou's contributions were mainly dressed in prose. He was, unlike his teacher Marshall, not well educated in mathematics. He went to Cambridge to study history and literature. A full fledged stringent version of modern welfare theory had to wait until the publication of Abba P. Lerner's (1934) paper and the book *The Control of Economic Resources* (1944). Lerner was the first to describe the system as a whole and to show that a competitive market economy generates a Pareto optimal allocation of resources; a result known as the *First Fundamental Theorem of Welfare Economics*. Starting from a competitive equilibrium he shows that the conditions for an optimal allocation of consumption goods are fulfilled, as well as the condition for efficiency in production. Finally, he shows that in equilibrium there is equality between the marginal rate of substitution in consumption and marginal rate of transformation in production for each pair of goods. The reason is that both producers and consumers optimize their action facing the same price vector. In other words, a competitive economy generates a Pareto optimal allocation of resources where both consumption and production are efficient; a formal proof of Adam Smith invisible hand conjecture. A similar proof can be found in Oskar Lange (1942), while Kenneth Arrow (1951a) uses topological methods and separating hyperplane theorems.

Lange and Taylor (1938) and Lerner (1944) also discussed the reverse result, that all Pareto optima can be supported by a price system after lump sum transfers of the initial wealth endowment. They did not produce a formal proof, but the conjecture was important for the discussion whether planned economies could reach a Pareto optimum. The first formal proof of this conjecture is probably due to Arrow (1951a), and the result is known as the *Second Fundamental Theorem of Welfare Economics*. We will return to these theorems in the following.

The Pareto criterion leaves the distributional problem unsolved. Arbraham Bergson suggested, in a paper published in 1938, that this problem can be addressed by a welfare function, which is an increasing function of the consumer's utility functions. Technically, we can now solve the resource allocation problem by maximizing the social welfare function subject to the technological constraints. The resulting allocation will be Pareto optimal, and the income distribution will be the appropriate one. However, the more specific preferences we build into the welfare function, the more relevant it will be to ask the question: Why this particular form of welfare function? Where does it come from, and does it reflect the preferences of the population in a reasonable way? This problem was approached by Arrow, who in a famous monograph first published in 1951 showed that if one starts from reasonable axioms on individual preferences and tries to aggregate them into a social ordering that fulfils similar axioms, this is impossible. At least one of the social choice axioms is violated. Most proofs (including Arrow's own) show this by proving that the non-dictatorship condition is violated. Many researchers have tried to modify the axioms to resolve the conflict between individual and social orderings, but no fully satisfactory solution has been found. The result is called *Arrows Impossibility Theorem* or, for that matter, the *Third Fundamental Theorem of Welfare Economics*.

The rest of the chapter is organized as follows. Section 2 sets out a simple static Walrasian general equilibrium model, which serves as a benchmark to be used in later sections. The principles of cost benefit analysis are dealt with in Sections 3. Section 4 addresses three fundamental issues; namely, The First and Second Welfare Theorems mentioned above, as well as introduces *The Core of the Market Economy*. The welfare gains from free trade are briefly discussed in Section 5, while Arrow's Impossibility Theorem is discussed in Section 6. Section 7 deals with externalities and introduces the concept of Pigouvian taxes, whereas public goods are dealt with in Section 8. Section 9 briefly discusses the area of Mechanism Design. Finally, Section 10 extends the benchmark model to a dynamic framework. This is interesting for at least two reasons. First, the extension allows us to address the time-dimension and, therefore, introduce dynamic analogues to some of the concepts addressed in earlier sections. Second, and more importantly, it enables to connect our survey on Welfare Theory to the growing literature on welfare measurement in dynamic economies. Section 11 gives a brief summary and some concluding comments.

2. A Simple Walrasian General Equilibrium Model

To illustrate the efficiency properties of a Walrasian equilibrium model, we will discuss the simplest Walrasian equilibrium model that involves production. We later modify the model to deal with distributional issues and the First- and Second Welfare Theorem, externalities, taxes and second best considerations. We also discuss Arrows Impossibility Theorem (The Third Welfare Theorem).

Let us start with the consumer, who has a strictly concave and twice continuously differentiable utility function denoted by

$$u = u(x, l^{s}) \tag{1}$$

where x is the demand for a consumption good, and l^{s} is the supply of labor. The utility function is increasing in consumption and decreasing in labor. The budget constraint of the consumer is

$$\pi + wl^{s} - px = 0 = B\left(x, l^{s}; p, w, \pi\right) \tag{2}$$

where p and w are the prices of consumer goods and labor. Here π is the profit income from the production sector/the firm. It enters the budget constraint, since the representative individual is assumed to own the firm. The firm has the technology/ production function

$$x^{s} = f(l) \tag{3}$$

where x^{s} is the supply of goods and l is the demand for labor/the input of labor. The production function is increasing in l, strictly concave and twice continuously differentiable.

The firm maximizes profit, while treating p and w as exogenous, which generates the optimal profit function

$$\pi[p, w)] = \max_{l}\{pf(l) - wl\} = px^{s}(p, w) - wl(p, w) \tag{4}$$

The consumer maximizes utility subject to the budget constraint, treating p, w and π as exogenous, which gives the optimal value (indirect utility) function

$$u\left[x(p, w, \pi), l^{s}(p, w, \pi)\right] = \max_{x, l^{s}}\left\{u\left(x, l^{s}\right)\middle| x, l^{s} \in B\left(x, l^{s}; p, w, \pi\right)\right\} \tag{5}$$

Now, let us substitute the profit function into the budget constraint as well as into the demand and supply function of the consumer to obtain

$$B(p,w) = p\left[x(p,w) - x^{s}(p,w)\right] + w\left[l(p,w) - l^{s}(p,w)\right] = 0 \qquad (6)$$

The first term is the value of excess demand in the market for goods, while the second term is the value of excess demand in the labor market. This equation means that *the values of excess demands sum to zero, independently of prices.* A little thought reveals that this condition holds for any number of markets and any number of consumers or firms. One can view Eq. (6) and its generalization as the economy's aggregate budget constraint, and the summation result corresponds to Walras' law. General equilibrium is typically defined as a situation, in which demand equals supply in all markets. Another, more stringent definition allows for excess supply in a number of markets, but in this case the equilibrium price in these markets must be zero. Eq. (6) shows that if one market is in equilibrium, then the other market must also be in equilibrium. This means, in the *n*-market case, that there are only n-1 independent markets but n prices. This is why Walras used one good as numeraire with a price equal to one. The trick works since the demand and supply functions are homogenous of degree zero, i.e., doubling all prices does not change firms' and consumers' optimal decisions. Firms and consumers lack money illusion.

Now, if we scale everything with the inverse of the price for goods, we can determine the relative price, $\omega = w/p$, either by equating the demand and supply of goods or by equating the demand and supply of labor. Say that we solve for the equilibrium price by equating demand and supply in the market for goods:

$$x(\omega^{*}) = x^{s}(\omega^{*}) \qquad (7)$$

where $\omega^{*} = w/p$ is the equilibrium real wage, which can be used to solve for all variables in general equilibrium.

In Figure 1 we illustrate the equilibrium in our Robison Crusoe Economy. On the vertical axis we measure the demand and supply of goods and on the horizontal axis the supply and demand of labor.

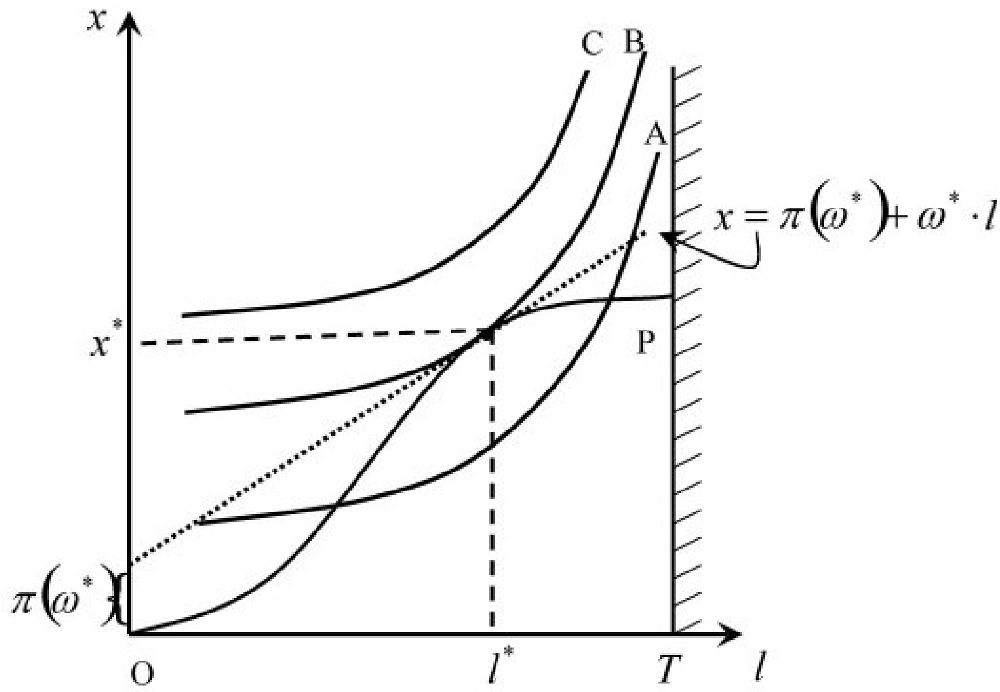

Figure 1: The Competitive Equilibrium

The concave production function is illustrated by the curve OP, and the convex indifference curves are denoted A, B, and C. The arrow in the diagram points out the direction of increasing utility. The curve denoted B has a common point with the production function, and through that point the straight line $y = \pi(\omega) + \omega l$ is tangent to both the indifference curve and the production function. This straight line corresponds to the economy's aggregate budget constraint. The slope of the line is the real wage rate in equilibrium.

In Figure 1, the competitive equilibrium is given by the point (l^*, x^*), which is assumed to be unique. However, uniqueness is not generally true, and an equilibrium may not even exist. For the equilibrium to exist in the case under consideration it is not necessary that the utility function is strictly concave. It is enough that the set above an indifference curve is a strictly convex set, which is the case in the figure.

It is also obvious from the geometry that the equilibrium point maximizes utility. We will, however, shortly move to the First and Second Welfare Theorems. To produce more general theorems, we need a more general model than a Robinson Crusoe Economy.

The Determination of the absolute price level

The reader may wonder how absolute prices are determined. Monetary theory is a well-developed branch of economics, and it would take us too far to go into any details. We briefly present a classical way to determine the price level called the Quantity Theory of Money. The underlying idea is that the quantity of money matters for nominal but not for real entities. The theory is extremely old. The economic writer that is considered to be the modern "Father of the Quantity Theory" is the Italian Benardo Davanzati (1588).

In the context of our simple general equilibrium model, the following equation system generates the nominal price- and wage levels:

$$wl^* + px^* = MV$$

$$\frac{w}{p} = \omega^*$$

The left hand side of the first equation is the total value of goods in circulation, i.e., wage income plus the nominal value of consumption. The right hand side contains the stock of money times the velocity of circulation. Both entities are exogenously determined. The second equation is the definition of real wage in general equilibrium. Entities which have a star for the index are equilibrium values that are determined in the real part of the economy. It is easy to show that

$$p = \frac{MV}{\omega^* l^* + x^*}$$

$$w = p\omega^*$$

Note that the prices are proportional to the quantity of money, and that increased money supply does not affect the real entities.

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