# APPLICATIONS OF GRAVIMETRY AND METHODS OF SURVEY

**Erwin Groten**
*Institute of Physical Geodesy, TU Darmstadt, Germany*

**Keywords:** gravimetry, gravity, gravitation, geoexploration, geodynamics, inertial navigation technology (INS).

**Contents**





**Summary**

Gravity and gravitation belong to the fundamental forces of physics. In the Newtonian sense, the “apple falling from the tree” illustrates a fundamental (horizon) system of directions in space, and yields the possibility, by measuring its location and the time it takes to fall, to determine the modulus (amount) of the gravity vector. Only a few sectors of life are not, in any way, affected by gravity. Consequently, the applications of gravimetry are numerous: navigation, various types of high-precision calibration, geoexploration, planetary exploration, satellite and deep space orbit determination, water level (particularly, mean sea level) studies, tides, all kinds of trajectories in space, time measurement and dissemination, geodynamics, precise determination of heights, elevations and their temporal variations, mining surveys, surveying and geodesy, and so on. They all depend, in one or another way, on the precise knowledge of gravity, that is, on the results of space and terrestrial gravimetry (and associated surveys) as obtained from borehole or various types of classical terrestrial gravimeters, as well as from space

(satellite) observations. DGPS (Differential GPS)-techniques have greatly improved all types of terrestrial gravity surveys. In addition, (kinematic) GPS, velocity determination, as well as less expensive and more efficient inertial navigation technology (INS) have made shipborne gravity surveys more reliable, and led to a complete revival of airborne gravimetric surveys in the early 1990s. Moreover, satellite altimetry strongly benefited from spaceborne GPS.

In principle, all near-Earth satellite techniques can also be applied to gravimetric lunar or planetary field studies; some of them have led (or can lead) to excellent information on the lunar and planetary (for example, Martian) gravity fields.

Whenever one relies on accuracy better than one part in a hundred million, one has to apply Einsteinian or general relativity theory instead of Newtonian concepts. The impact of space observations is strongly increasing; consequently, various types of satellite measurements gain more interest where gravity is often a by-product that is obtained at relatively low costs.

## 1. Introduction

Almost all phenomena on Earth are, in one or the other way, influenced and affected by Earth's gravity field. In many cases, a precise determination and evaluation of gravity is necessary; typical examples are satellite orbits, calibration procedures, navigation, and precise elevation determination.

For most practical applications, and in view of the dependence of gravity on topography, it would be a reasonable goal that gravity surveys should provide (in developed countries) gravity with accuracy of $0.2 \times 10^{-5}$ m s$^{-2}$ (or 0.2 milliGal (mGal) where 1 Gal corresponds to $10^{-2}$ m s$^{-2}$) at station separation of 2 km to 5 km. Elevations for DTM (digital terrain models) with grid distances of no more than 40 m should also be available. In general, accuracies of $0.5 \times 10^{-5}$ m s$^{-2}$, referred to precisely defined frames of reference, with wavelengths of about 10 km, could be sufficient.

Correspondingly, absolute gravity measurements should provide reference stations with accuracies of 10 μGal (microGal) and 3 μGal precision at station separation of several hundred kilometers, so that relative gravimetry can fill in by densification. For special purposes (tides, geodynamics, geoexploration), higher local accuracy may be needed. In developing countries, we are still far apart from this situation. This aim can only be achieved by the combined use of space (satellite), terrestrial stationary, and mobile (air- and shipborne) gravimetric approaches; moreover, gravity surveys are also needed at sea, on ice and islands, in deserts, in space, and around the Moon and planets. Present space projects such as GOCE and GRACE aim for global coverage at a wavelength of about 100 km with about 1 mGal accuracy. GRACE puts emphasis on low harmonics, whereas GOCE emphasizes higher harmonics.

Mobile terrestrial techniques, in combination with space techniques, should deliver in the future mean gravity values for wavelengths at Earth's surface (or blocks) of 5 km to 10 km with accuracies better than 0.1 mGal, which is meanwhile more important than point gravity values with higher accuracy. Accordingly, DTM for grid distances of 20 m is desired, in general, for the future.

In gravimetry, we basically determine the modulus $g = |\vec{\mathcal{G}}|$ of gravity at a geocentric location $P(x_i)$ within a topocentric horizon system $(x_i')$ for a "tide-free" Earth model where, however, the permanent tides (corresponding to $M_o$ and $S_o$, the permanent lunar and solar tides, respectively) are not fully eliminated. Permanent tides need a special treatment in view of their anharmonic part. However, the direction $\vec{e} = \vec{\mathcal{G}}/g$ (with $g = |\vec{\mathcal{G}}|$) of the gravity vector $\vec{\mathcal{G}}$ is also observed using long vertical and horizontal pendulums for special purposes. Moreover, the direction $\vec{e}$ with respect to an ellipsoidal geocentric reference system is derived in terms of plumb line or vertical deflections from *g* using Vening-Meinesz's formula. Deflections of the vertical are used for various geophysical and geodetic purposes. In gravity gradiometry, we aim at accuracies of about one Eötvös (E), where 1 E = $10^{-9}$ $s^{-2}$, mainly in space applications. Single integration then yields (relative) gravity, and double integration leads to potential differences where the geopotential *W* is usually expressed in kGal m and 1 kGal m = 10 $m^2$ $s^{-2}$. Gradients represent a tensor with components $\partial^2 \mathcal{W} / x_i' \partial x_j' \left(i, j = 1, 2, 3\right)$ in a Cartesian topocentric terrestrial system, and second derivatives are usually abbreviated by $W_{ij}$. In space applications, an orbital Cartesian system is preferred, with radial, cross-track, and along-track components as described below.

## 2. Gravity Representation of the Deformable Earth and its Models

The gravity field of the deformable Earth is built up by gravitation, that is, the attraction of the solid and fluid Earth, its atmosphere, and the attraction of the masses in space. On a rotating Earth, the centrifugal potential $\mathcal{H}$ also has to be considered. Due to the mobility of those masses, the gravity field varies with time. If we ignore, at first, the mass of the atmosphere (which amounts to only $10^{-6}$ of the fluid and solid masses of Earth itself), and dismiss for a while extraterrestrial masses (planets, Moon, Sun etc.) one may associate a scalar gravity potential $\mathcal{W}$ with the gravity acceleration $\vec{\mathcal{G}}$ by the relation (at any time *t*):

$$-\vec{\mathcal{G}} = \nabla \mathcal{W} \tag{1}$$

where the minus sign is a matter of convention depending on the definition of work associated with the potential. In an "earth-fixed" geocentric coordinate system, $x_i$, where the $x_3$-axis points towards the geographic North Pole and $(x_1, x_2)$ define a terrestrial equator, and where $x_1$ lies in the Greenwich meridian (plane), at any location $P$ the gravity vector $\vec{\mathcal{G}}$ points towards the zenith. The plane orthogonal to $\vec{\mathcal{G}}$ is called the horizon, and may define at $P$ a topocentric coordinate system $x_i'$ with $x_3'$ antiparallel to $\vec{\mathcal{G}}$, $(x_1', x_2')$ defining the plane of horizon, with $x_1'$ pointing towards the north and $x'$ pointing towards the east. The transition between $x_i$ and $x_i'$ is done by well-known relations illustrated by the "astronomical triangle." As $x_i$ is rotating with the velocity of Earth, $\vec{\omega}$, where $x_3$ is however not parallel to $\vec{\omega}$ (in view of polar motion $(\vec{\omega}, x_3)$ which amounts to about 10 m at the pole), we further need a geocentric inertial frame, $X_i$, which is considered to be "space-fixed," with $X_3$ aligned to $\vec{\omega}$ and $(X_1, X_2)$ defining the celestial equator $\vec{\omega}$ (ignoring small nearly-diurnal rotations with amplitudes of ~0.2 m which make up the difference between $\vec{\omega}$ and the direction from geocentre to the Celestial Ephemeris Pole (CEP)); $X_1$ points towards the equinox, which defines the intersection between the celestial equator and the ecliptic being the path of the Earth–Moon barycentre around the sun. Due to nutation and precession, $X_i$ is not a perfect inertial system in the Newtonian sense; it, therefore, needs corrections.

Eq. (1) basically defines a "conservative" force, where a time-dependent $\mathcal{W}(\vec{r}, t)$ has to be reduced to $\mathcal{W}(\vec{r})$, which depends only on the geocentric location $\vec{r}(x_i)$. This means that a "tide-free" model-Earth replaces the actual Earth. The gravitational effect of the atmosphere is taken into account by "atmospheric corrections" where, however, effects of the order of $10^{-7}$ can often be ignored. The basic unit of gravity, *g*, (being the modulus of $\vec{\mathcal{G}}$) is the milliGal (= $10^{-5}$ m s$^{-2}$), where tidal effects are of the order of 0.2 mGal at Earth's surface. $\mathcal{W}$ is built up by the potential of gravitation, $\mathcal{V}$, plus the (anharmonic) potential of the centrifugal acceleration, $\mathcal{H}$ where the centrifugal force amounts at Earth's surface to ~1 % of *g*; consequently:

$$\mathcal{W} = \mathcal{V} + \mathcal{H} \tag{2}$$

In a vacuum, $\mathcal{V}$ fulfills the Laplace equation so that:

$$\Delta\mathcal{V} = 0 \tag{3}$$

which is a special case of the Poisson equation:

$$\Delta\mathcal{V} = -4\pi G\rho \tag{4}$$

where $\rho$ is density and $G$ the Newtonian gravitational constant. For global purposes, $\mathcal{V}$ may be expanded, as a harmonic function, in a series of spherical harmonics (where $(\varphi, \lambda)$) are latitude and longitude, respectively). At Earth's surface, we get an asymptotic series yielding good approximations in the form (with geocentric spherical coordinates $(r, \varphi, \lambda)$)):

$$\mathcal{V}(r, \lambda, \varphi) = \frac{GM}{r} + \sum_{n=1}^{\infty} \sum_{m=0}^{\mathrm{n}} \left(a_n^m \cos m\lambda + b_n^m \sin m\lambda\right) P_n^m(\theta) / r^{n+1} \tag{5}$$

with colatitude $\theta = 90° - \varphi, P_n^m$ are Legendre polynomials of degree $n$ and order $m$ and $M$ is the mass of Earth. A more convenient form of Eq. (5) reads (with $n'$ = degree of approximation, truncation, and $R$ = equatorial radius):

$$\mathcal{V} = \frac{GM}{r} + \sum_{n=1}^{n'} \sum_{m=0}^{n} \left(\frac{R}{r}\right)^{n+1} \left(C_n^m \cos m\lambda + S_n^m \sin m\lambda\right) P_n^m(\theta). \tag{5a}$$

With $\vec{\mathcal{G}}$ being tangent to the plumb-line at any point $P$, we obtain in spherical approximation (which means an approximation of the order of the flattening of Earth or 1/300):

$$-g \cong \partial V/\partial r \tag{6}$$

A spheroid (rotational ellipsoid) is considered as a good approximation to Earth if it rotates with mean $\vec{\omega}$ (within $\pm 10^{-8}$), has its origin at the geocenter, has semi-major axis $a = R$, with $a \cong 6378136.7\,\text{m}$, flattening $(a-b)/a \cong 1/298.25$ where $b =$semi-minor axis, and its mass is equal to $M$. We may define such an earth-ellipsoid (in the best fitting sense with respect to the geoid) as a "normal Earth" so that its "normal" potential is denoted by $\mathcal{U}$, $-\vec{\gamma} = \nabla \mathcal{U}$, and the geoid is (approximately) coinciding with a level surface $\mathcal{W}$ = constant at mean sea level (MSL) all over Earth. Since the unit of $\mathcal{W}$ ($m^2\ s^{-2}$) is only dependent on time (because the meter is basically now defined by the time of propagation of light in a vacuum), there is no scaling problem in (absolute) gravity measurements. Determinations of *GM* and its temporal variations, if any, are extremely important, as *GM* is a "scale factor" in space science, similar the role of *G* in terrestrial science. *GM* in Eq. (5) is observed from satellite orbits according to Kepler's third law, and is, therefore, known with extremely high accuracy, whereas *G* has only been determined until now with an accuracy of less than $\pm 10^{-6}$ based on the Cavendish torsion balance principles; modern modifications did not yet yield substantially higher accuracy. Space experiments may yield better precision in the future. Consequently, the mass of Earth is still not yet known any. From the (harmonic) disturbing potential:

$$\mathcal{T} = \mathcal{W} - \mathcal{U} - \tag{7}$$

we find the separation between the earth ellipsoid and the aforementioned geoid, $N$ (geoid height), using Bruns' formula:

$$N = \mathcal{T}/\gamma\,. \tag{8}$$

and the angle $\left(\vec{\mathcal{G}}, \vec{\gamma}\right)$ is called deflection of the vertical. However, Eq. (8) holds only if the potential $\mathcal{U}$ at the ellipsoid is exactly equal to $\mathcal{W}$ at the geoid, which now holds approximately within $10^{-7}$. We may define the gravity disturbance at any point P (with outer ellipsoidal surface normal *n*):

$$\delta g(P) = g(P) - \gamma(P) = -d\mathcal{T}/dn \tag{8a}$$

whereas the difference:

$$\Delta g(P) = g(P) - \gamma(Q) = \delta g(P) + N d\gamma/dn \tag{8b}$$

is called the gravity anomaly, and $Q$ is the vertical projection (map) of any point $P$ of the geoid onto the ellipsoid.

The theory and practical formulation of equipotential reference gravity models in addition to the original Somigliana-Pizzetti ellipsoidal model for normal gravity $\vec{\gamma}$ are presently under consideration. Truncated higher order spherical harmonic expansions could also be one alternative for global geophysical modeling. Theoretical aspects of this kind are discussed in more detail elsewhere (see chapter *Gravity Field of the Earth*). A more reliable value for the geopotential at the geoid is $W$ = 62 636 856.0 $m^2 s^{-2}$.

By ignoring first the other force fields besides Earth's gravity field, we may, in a Newtonian inertial geocentric system $X_i$ (reduced for nutation and precession), write the equation of motion:

$$\vec{\mathcal{F}} = m\vec{a} \tag{9}$$

where $-\vec{a} = \nabla\mathcal{V}$, $m$ is a mass point, and $\vec{\mathcal{F}}$ is any force. Then, for $m = 1$ we find for a multiple force field:

$$\sum_j \vec{\mathcal{F}}_j \equiv \sum_j \left\{ f_{ij} \right\}$$

$$\ddot{X}_i = \sum_j f_{ij} \tag{10}$$

and, again, for $\vec{\mathcal{G}} = \left\{ g_i \right\} \left( i = 1,2,3 \right)$ and $\vec{a} = \left\{ a_i \right\}$, (where $g_i$ and $\nabla\mathcal{V}_i$ are projections of $\vec{\mathcal{G}}$ and $\nabla\mathcal{V}$ onto $X_i$, respectively). If we ignore non-gravitational forces:

$$\frac{d^2 X_i}{dt^2} \equiv \ddot{X}_i = -a_i = \nabla\mathcal{V}_i \quad \text{and, analogously,} \quad \frac{d^2 x_i}{dt^2} \equiv \ddot{x}_i = -g_i \tag{10a}$$

so that double integration with respect to time yields coordinate differences $X_i\left(t\right) - X_i\left(t_0\right)$.

Because of the varying density distribution in the atmosphere (passat and monsoon wind, atmospheric "lows" and "highs," associated loading at Earth's surface, etc.), in the oceans, and in the solid earth, the location of the geocenter is not perfectly stationary within the earth; it varies with subdecimeter changes. If it were exactly stationary, the first degree terms in Eq. (5) would disappear in a geocentric system. In addition, the terms of degree two and order one would vanish if the terrestrial (geographic) pole were exactly located on the axis of maximum moment of inertia (figure axis).

In any non-inertial system related to the rotating Earth there are the centrifugal acceleration:

$$\vec{c} = \vec{\omega} \times \left( \vec{\omega} \times \vec{r} \right) \tag{11}$$

and the Coriolis acceleration:

$$2\left( \vec{\omega} \times \vec{v} \right) \tag{12}$$

where $\vec{v}$ is the velocity of a mobile station within an "earth-fixed" frame. If, in addition, Earth's spin vector $\vec{\omega}$ is considered as time varying rotation with $\vec{\omega} \equiv d\vec{\omega}/dt \neq 0$, there is also the Eulerian acceleration:

$$\vec{\omega} \times \vec{r} \tag{13}$$

The accelerations in Eqs. (11) and (12) play a significant role for gravimetry in (mobile) vehicles such as airplanes, ships, and boats. Thus, velocities have to be taken into account in exactly defined mobile frames of reference that can be implemented by three-axes gyroscope systems, to which GPS (Global Positioning System) satellite receivers and/or accelerometers can be related on-board. In case of navigation, these observations have to be processed in real-time. In this way, the precise transformations between the aforementioned reference frames become important. Accelerations within the solar or galactic system are seldom taken into account. However, recently, for exact tidal corrections, the nearest (terrestrial) planets and luni-solar effects have been taken into account. In this context, a (partly anharmonic) tidal potential is derived from the tidal acceleration, which is basically the difference between the gravitational acceleration at a station P and the orbital acceleration (of Earth in its orbit around a celestial body) at the same point. On a deformable body (such as Earth) rotating around the Sun there is a triple tidal effect consisting of (a) the attraction of the celestial body, (b) the shift of station P due to the deformation of Earth's surface at P, and (c) the potential change due to the deformation (mass redistribution). In addition, on an Earth covered by oceans, the shift of water masses generates (a) a loading effect (deformation), (b) varying attraction of redistributed masses, and (c) the shift of the deformed earth surface caused by the load. Actually, there are consequently six tidal contributions. Also the (tidal) vertical acceleration of the deformable surface of Earth at a station P generates a small additional tidal (inertial) perturbation; however, it is only considered in very precise tidal work. If we define the earth ellipsoid as a level ellipsoid, with $U°$ (= $U$ on the ellipsoid) constant, it is uniquely defined by four independent parameters such as ($f$, $a$, $\omega$, $GM$); we may also replace, for example, $f$ by the second degree zonal harmonic of the gravitational potential.

In general, gravity is treated within the terrestrial system as a Newtonian quantity, so that relativistic corrections play only a significant role in extraterrestrial considerations. These orbital calculations are usually carried out in a Newtonian inertial reference frame. Relativistic corrections are taken into account with satellite positioning in the special relativistic sense and in corrections of time, in view of potential differences between clock locations and the potential $\boldsymbol{W}$ at the geoid, and in deriving TAI (temps atomique international, or international atomic time) in the general relativistic sense. Also, with the reduction of long range distances obtained from VLBI (very long base line interferometry), lengths are treated in the relativistic sense. Beyond these effects, the gravity field is evaluated in the Newtonian sense.

We may conclude this introductory section with some summarizing remarks on the aforementioned reference systems. The topocentric (astronomical) horizon system oriented along $\vec{\mathcal{G}}$, north and east, can be identified with the "observation space" of terrestrial gravity measurements where $\vec{\mathcal{G}}$ is implemented by a bubble or electronic level (or even a vertical pendulum). The associated "model space" is usually the (geodetic) horizon system, where the role of $\vec{\mathcal{G}}$ is played analogously by the (unique) ellipsoidal normal, $\vec{\mathcal{N}}$, which slightly differs in direction from $\vec{\gamma}$. The transition between both systems is done by the Laplace condition, which takes into account the small differences between astronomical north (meridian) and geodetic north (geodetic meridian), as well as astronomical and geodetic horizons.

For space observations, the transition from the non-rotating, "pseudo-inertial" system $X_i$ to the "earth fixed" system $x_i$ is done using Greenwich (apparent sidereal) time, being basically the Eulerian angle between $x_1$ and $X_1$. For satellite positioning, we may use Keplerian elements ($a$, $i$, $e$, $\Omega$, $\omega$, $v'$) as an alternative to Cartesian coordinates, where the six elements correspond to $\left(X_i, \dot{X}_i\right)$; they are explained in any textbook on (Keplerian) orbits. Even if we ignore galactic rotation and the accelerations of the Earth–Moon system in the ecliptic (including planetary precession), the luni-solar precession-nutation of $X_i$ has to be taken into account in evaluating satellite locations (ephemerides) in space. We may also apply a rectangular topocentric orbital space system with origin at the spacecraft-mass center with along-track, cross-track, and radial axes or components.

Similarly, the earth-fixed (rotating) equatorial system, $x_i$, is not really "earth-fixed," as the lithosphere (uppermost, at depth < 100 km, "rigid" plate system at Earth's surface) is moving horizontally in an almost monotonous way at velocities of some centimeters per year, with collision and subduction zones due to individual motion of plates (for details see chapter *Tectonic Processes*). The terrestrial equatorial system may be identified with the "model space," and the associated (rotating) equatorial system oriented along the Celestial Ephemeris Pole (CEP) would play the role of the "observation space." The three Eulerian angles for the transition from terrestrial to celestial systems would then be the Greenwich-meridian (longitude zero) component of polar motion, its component rectangular to it in the plane tangential to the terrestrial (geographic) pole and Greenwich time, as described above.

-
-
-

## Bibliography

Altiner Y. (1999). *Analytical surface deformation theory*, 100 pp. Berlin: Springer Verlag. [Global and large-scale deformations, as derived from satellite methods, are treated in this booklet.]

Bursa M. and Kostelecky J. (1999). *Space Geodesy and Space Geodynamics*, 460 pp. Prague: Ministry of Defense, Czech Republic. [This book gives an up-to-date general review of global gravity field aspects.]

Groten E. (1979/80). *Geodesy and the earth's gravity field*, Vols. I and II, 724 pp. Bonn: Duemmler Verlag. [The two volumes represent an introduction to the theory and application of the determination of the gravity field of Earth.]

Hawking S. W. and Israel W. (1987). *Three hundred years of gravitation*, 684pp. Cambridge: Cambridge University Press. [This book summarizes the history of the theory of gravitation up to today.]

Lambeck K. (1988). *Geophysical Geodesy*, 718 pp. Oxford: Clarendon Press. [This book is a general introduction to geodynamics with emphasis on geodetic aspects.]

Melchior L. (1983). *The tides of the planet earth*, Second edition, 641 pp. Oxford, UK: Pergamon Press. [The time-variable gravity field, with emphasis on tides, is elucidated in this book.]

Moritz H. (1980). *Advanced physical geodesy*, 500 pp. Karlsruhe: H. Wichmann Verlag. [The theory of the gravity field is discussed with respect to advanced applications.]

Telford W. M., Geldart L. P., Sheriff R. E., and Keys D. A. (1976). *Applied Geophysics*, 860 pp. Cambridge: Cambridge University Press. [A general introduction to geoexploration, including gravimetric techniques, is given in this book.]

## Biographical Sketch

**Erwin Groten** was born in 1935, and education in geodesy and geophysics in Munich, Germany. After professional activities, Ph. D. in geodesy in 1963, research associate at Ohio State University, and teaching activities at Technical University of Munich. Since 1972, full professor of geodesy at Darmstadt Institute of Technology. Honorary professor of Universities of Wuhan and Shanghai. Various research and teaching activities in Germany and abroad. Former President of Section V of International Association of Geodesy, President of National Committee of Geodesy and Geophysics in Germany (1995–99), member of various Academies of Sciences, of professional institutions, and international organizations.