NUCLEAR REACTOR THEORY

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Contents

Summary

Some of the neutrons produced by fission in a nuclear reactor ultimately cause further fissions to maintain the nuclear chain reaction. Most of the remaining neutrons are absorbed in non-fissile material while some are lost by leakage from the reactor. The paths of the neutrons during this brief period are all important in the design and operation of a nuclear reactor.

Neutrons in a reactor diffuse in all directions and, due to the very small space occupied by the nucleus of each atom, are not constrained to move in any particular direction. Their behavior matches that described by Fick's law and the net flow is down the concentration gradient. During the diffusion process many scattering collisions occur causing some individual neutrons to move up the concentration gradient following random scattering. During the diffusion process some neutrons are absorbed and any which eventually cross the reactor boundary or surface are lost by leakage. This leads to the steady state diffusion equation which, in simple terms, states that neutron production is equal to neutron absorption plus neutron leakage for stable conditions to be maintained.

For a reactor of infinite size, where no neutron leakage is possible, neutron production is equal to neutron absorption. This is defined mathematically by the four factor formula. In this formula each of the four factors gives the average value by which the number of neutrons must be multiplied due to induced fission (reproduction factor), fast fission (fast fission factor), resonance absorption (resonance escape probability), and neutron absorption in reactor materials (neutron utilization factor). The first two factors are production factors and are greater than unity while the latter two are absorption factors and are less than unity. For steady state conditions all factors multiplied together must be unity, that is, the number of neutrons in any generation is equal to the number in the previous generation.

For a reactor of finite size, neutron leakage factors must be introduced resulting in the six factor formula. Since these factors, actually non-leakage factors, are less than unity, the reproduction factor or the neutron utilization factor must be increased to respectively increase neutron production or reduce neutron absorption. Since neutron leakage depends upon the surface area of the reactor while neutron production (and absorption) depends upon the volume of the reactor, it is evident that, as the reactor linear dimensions are decreased, the surface area becomes more significant relative to the volume and the leakage will ultimately overwhelm production plus absorption. This defines the minimum critical size for a nuclear reactor to maintain a fission chain reaction.

A further consequence of neutron leakage is that, for uniform neutron production throughout the reactor, leakage from the surface will result in a lower concentration of neutrons near the surface of the reactor since no neutrons diffuse back into the reactor. Theoretically, this results in a sinusoidal variation of neutron concentration across a reactor of finite dimensions. In a rectangular (cubical) reactor, this variation is in three dimensions so the average neutron concentration is only about one quarter of that in the centre. Since heat production and power output are directly related to the neutron concentration, it is desirable to create a more uniform concentration across the whole reactor. All practical nuclear reactors therefore employ methods for doing this.

1. Neutron Diffusion Characteristics

1.1. Basic Concepts

In order to appreciate the diffusion of neutrons through a medium it should be noted that atoms consist mainly of empty space. The tiny nucleus occupies an almost negligible volume but has virtually all the mass of the atom while the electron cloud surrounding the nucleus has almost negligible mass. Uncharged neutrons which are very heavy compared

with the electrons can pass freely through the cloud of negatively charged electrons. Neutrons do interact with the nuclei of the atoms but, since the nuclei occupy so little space, the chances of interaction are extremely small. However in passing through a very large number of atoms the neutrons will eventually strike a nucleus or come close enough for an interaction to occur. In a two dimensional situation the trajectory of a neutron is analogous to that of a golf ball being hit into a forest of trees. The chance of it hitting any single tree is minimal but if it travels far enough it will eventually hit one.

A further important principle is that neutrons will interact more readily with nuclei when moving at a lower velocity. This is particularly important if the interaction leads to fission and the continuation of a fission chain reaction. Such a reaction therefore is promoted by reducing the kinetic energy and hence the velocity of the neutrons. This can be achieved by multiple scattering collisions with nuclei. A certain amount of energy is lost with each collision until the neutrons reach a state of thermal (energy) equilibrium with the nuclei of the medium. Although absorption of neutrons generally leads to fission in fissile materials it can also lead to radiative capture. Hence there are three important interactions to consider: scattering, absorption and fission. For any material there is a certain probability of each occurring. This is expressed as the microscopic cross section of the material. It is the effective projected area of the nucleus and designated as σ.

σ_s scattering (elastic or inelastic) cross section

σ_a absorption (radiative capture) cross section

σ_f fission cross section

To promote a reduction in neutron energy, so as to enhance subsequent fission in a fissile material, a large scattering cross section is desirable. To minimize the loss of neutrons during the scattering process a small absorption cross section is required. To maintain the fission chain reaction a high fission cross section is required. Fissile materials in which fission can occur have heavy nuclei which are not effective in reducing neutron energy so a separate material must be employed as a slowing down medium or moderator. This moderator must have a high scattering cross section and a low absorption cross section. Furthermore it should have a high logarithmic mean energy decrement (due to a low atomic mass number) so that the neutron energy is reduced adequately in a small number of collisions.

The foregoing concepts lead to the fundamental design of a nuclear reactor. Fuel and moderator are dispersed in such a way throughout the reactor core that neutrons produced by fission in the fuel pass through a moderator before re-entering the fuel at a lower energy to cause the next round of fissions. Within the moderator the neutrons suffer many scattering collisions which reduce their energy sufficiently for fission to occur readily in the fuel. While being scattered in the moderator as few as possible should be lost by absorption. This chapter deals with the passage or diffusion of neutrons through a scattering and absorbing medium.

1.2. Fick's Law

Returning to the two dimensional analogue of a golf ball striking trees in a forest it is

evident firstly that, after two or three collisions, the ball could be traveling in any direction and secondly that multiple balls undergoing the same process will have no effect on one another. The same applies to neutrons in a medium where diffusion takes place in all directions and the diffusion of those going in one direction does not affect those going in a different direction. If considered to be moving in a completely random manner, the neutrons diffuse in the same manner as the molecular diffusion of one gas within another. This diffusion may thus be described in the same way as chemical diffusion. Fick's Law describes such diffusion and states that, if the concentration of a solute is greater in one region than in another, the solute will diffuse from the region of higher concentration to a region of lower concentration. In the same way neutrons diffuse down a concentration gradient in a medium. In one direction (*x*-direction) this may be described mathematically as follows, where J_x is the neutron flow in the *x*-direction:

$$J_x = -Dd\phi / dx \qquad (1)$$

The neutron flux ϕ is of course the number of neutrons n multiplied by their velocity v

$$\phi = nv \qquad (2)$$

The flow J_x is thus proportional to the negative gradient of the number of neutrons n (or flux ϕ). The factor D is known as the *diffusion coefficient.*

Note that the motion of the neutrons is completely random and that they diffuse in all directions as they are scattered by collisions with nuclei. Some will invariably be going in the opposite direction to others. At any location x along the x-direction neutrons will be scattered and will diffuse both up and down the concentration gradient. Since there is a greater number of neutrons to the left (up the concentration gradient) than to the right (down the concentration gradient) more will be scattered and more will diffuse away from the location to the left. Thus more will diffuse down the concentration gradient than up the concentration gradient.

In three dimensions the same situation applies and the flow of neutrons is given by:

$$J = -D\text{grad}\phi$$

$$J = -D\nabla\phi \qquad (3)$$

Here ∇ is the gradient operator.

When neutrons strike a heavy nucleus in an elastic scattering collision, those making near head-on collisions rebound backwards and those making glancing collisions pass on in roughly the same direction. The net result is uniform scattering in all directions. However, when neutrons strike a very light nucleus, such as hydrogen, in an elastic collision, those making near head-on collisions are almost brought to rest as the nucleus gains forward velocity. The result is very little back scatter and increased forward scatter of the neutrons. This anisotropic scatter affects the mean free path λ of the neutrons which is stretched in

the forward direction. To account for this the *transport mean free path* λ_{tr} is used. This is the *scattering mean free path* λ_s corrected for the preferential forward scatter. Since the degree of increased forward scatter depends upon the mass of the nucleus it would be expected that the atomic mass number would appear in the correction formula. If the observed scattering angle is ψ then $\cos\psi$ can be determined for each of a large number of collisions. The average of this value is μ.

$$\mu = (\cos\psi)_{\text{average}}$$

For uniform scattering in all directions μ is equal to zero but for anisotropic or forward scatter it can be shown that μ depends upon the atomic mass number A as follows:

$$\mu = 2/(3A)$$

The relationship between the macroscopic transport cross section and the macroscopic scattering cross section is as follows:

$$\Sigma_{tr} = \Sigma_s(1-\mu)$$

Hence it is evident that, as the atomic mass number increases, μ becomes smaller and Σ_{tr} approaches Σ_s indicating less preferential scatter in the forward direction.

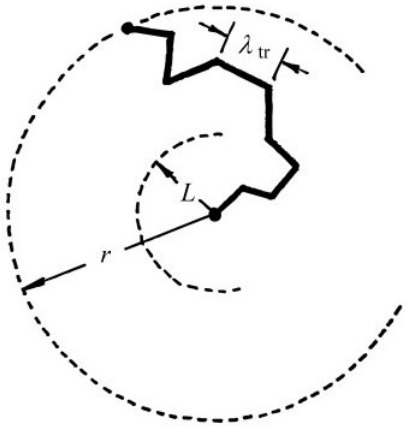
The transport mean free path is given by the following as is the case for the scattering mean free path:

$$\lambda_{tr} = 1/\Sigma_{tr} \qquad (4)$$

The diffusion coefficient D as defined above is given approximately as one third of the transport mean free path Σ_{tr}

$$D = \lambda_{tr}/3 \qquad (5)$$

From the above it can be seen that, knowing the mass number A of the medium and its macroscopic scattering cross section Σ_s the diffusion coefficient D may be calculated. These parameters as applied to neutron diffusion are illustrated in Figure 1.



$r = 2.4\,L$ (Direct migration distance)

$r^2 = 6\,L^2$

$L^2 = D / \Sigma_a$ (Diffusion length)

$D = \lambda_{tr} / 3$ (Diffusion coefficient))

$\lambda_{tr} = 1 / \Sigma_{tr}$ (Transport mean free path)

$\Sigma_{tr} = \Sigma_s (1 - \mu)$ (Macroscopic transport cross section)

$\mu = 2 / (3\,A)$ (Average cosine of scattering angle)

Figure 1: Neutron diffusion parameters

Fick's Law is however not an exact relation when applied to the diffusion of neutrons since some are absorbed within the medium. It however can be used when considering most reactor moderating systems which by design are weak neutron absorbers. In particular circumstances Fick's Law cannot be applied as it would give false results. Such circumstances are:

- In a medium that absorbs neutrons strongly
- Close to a neutron source
- Near the surface of the medium
- In a medium having a beam of neutrons

If neutrons are absorbed strongly, they are removed from the system and the equation will overestimate the neutron flux. Near the surface of the medium, neutrons diffusing out will not return and again the equation will overestimate the neutron flux. Close to a neutron source, the neutrons will be moving primarily in one direction rather than being scattered in all directions. In a beam of neutrons, neutrons will also be moving in one direction and the few that are scattered will not represent true diffusion in the medium.

Nevertheless Fick's Law may be used in many preliminary calculations regarding the flow of neutrons within a given volume.

2. Neutron Diffusion Equation

2.1. Neutron Balance

In an arbitrary volume of a certain medium containing neutrons, the change in the number of neutrons will be equal to the production minus the loss due to absorption and loss due to leakage.

$$Change = \text{Pr}oduction - Absorption - Leakage$$

This is usually expressed in terms of rate of neutron production and rates of absorption and leakage to give a rate of change.

The rate of production of neutrons $R_{\text{production}}$ depends upon the fission macroscopic cross section Σ_{f} and the neutron flux ϕ as well as on the number of neutrons produced in each fission. For the present analysis the rate of production will simply be designated as S

$$R_{\text{production}} = S \qquad (6)$$

The rate of absorption of neutrons $R_{\text{absorption}}$ depends just on the absorption macroscopic cross section Σ_{a} and the neutron flux ϕ.

$$R_{\text{absorption}} = \Sigma_{\text{a}}\phi \qquad (7)$$

It is evident from the above that the units of these equations are neutrons $m^{-3}s^{-1}$.

The rate of leakage is more difficult to determine. Fick's Law indicates that the flow of neutrons in a one dimensional situation is proportional to the negative gradient of the neutron concentration. Recall Eq.(1) and consider a three dimensional situation in Cartesian co-ordinates, a small rectangular prism of dimensions dx, dy, and dz. The flow of neutrons L across each face can be determined as follows. For one set of faces opposite one another in the x direction:

$$L_{\text{in}\,x} = J_x dydz = -D(\partial\phi/\partial x)dydz$$

$$L_{\text{out}\,x+dx} = J_{x+dx}dydz = -D[(\partial\phi/\partial x) + \{\partial(\partial\phi/\partial x)/\partial x\}dx]dydz$$

$$L_{\text{net}\,x} = L_{\text{out}\,x+dx} - L_{\text{in}\,x} = -D[(\partial\phi/\partial x) + \{\partial(\partial\phi/\partial x)/\partial x\}dx]dydz - D(\partial\phi/\partial x)dydz = -D(\partial^2\phi/\partial x^2)dxdydz$$

Similarly for the other faces:

$$L_{\text{net}\,y} = -D(\partial^2\phi/\partial y^2)dxdydz$$

$$L_{\text{net}\,z} = -D(\partial^2\phi/\partial z^2)dxdydz$$

The total leakage therefore is given by:

$$L_{\text{total}} = -D[(\partial^2\phi/\partial x^2) + (\partial^2\phi/\partial y^2) + (\partial^2\phi/\partial z^2)]dxdydz$$

On a per unit volume basis this reduces to the following where ∇^2 is the Laplacian Operator.

$$L^*_{\text{total}} = -D\nabla^2\phi \qquad (8)$$

The basic neutron balance equation where each term is expressed in terms of neutrons per unit volume per second can now be written as

$$Change = Production - Absorption - Leakage$$

$$Change = S - \Sigma_a\phi + D\nabla^2\phi$$

Under steady state conditions the change in neutron density is zero and the *steady state diffusion equation* is obtained.

$$D\nabla^2\phi - \Sigma_a\phi + S = 0 \qquad (9)$$

This equation may be rewritten in the following form:

$$\nabla^2\phi - (\Sigma_a / D)\phi = -S / D$$

Another parameter, the *diffusion length* L, may now be defined as follows:

$$L^2 = D / \Sigma_a \qquad (10)$$

The steady state diffusion equation may thus be written as

$$\nabla^2\phi - \phi / L^2 = -S / D \qquad (11)$$

Here S is a source within the diffusing medium and has units of neutrons/m^{3}s.

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Biographical Sketch

Robin Chaplin obtained a B.Sc. and M.Sc. in mechanical engineering from University of Cape Town in 1965 and 1968 respectively. Between these two periods of study he spent two years gaining experience in the operation and maintenance of coal fired power plants in South Africa. He subsequently spent a further year gaining experience on research and prototype nuclear reactors in South Africa and the United Kingdom and obtained M.Sc. in nuclear engineering from Imperial College of London University in 1971. On returning and taking up a position in the head office of Eskom he spent some twelve years initially in project management and then as head of steam turbine specialists. During this period he was involved with the construction of Ruacana Hydro Power Station in Namibia and Koeberg Nuclear Power Station in South Africa being responsible for the underground mechanical equipment and civil structures and for the mechanical balance-of-plant equipment at the respective plants. Continuing his interests in power plant modeling and simulation he obtained a Ph.D. in mechanical engineering from Queen's University in Canada in 1986 and was subsequently appointed as Chair in Power Plant Engineering at the University of New Brunswick. Here he teaches thermodynamics and fluid mechanics and specialized courses in nuclear and power plant engineering in the Department of Chemical Engineering. An important function is involvement in the plant operator and shift supervisor training programs at Point Lepreau Nuclear Generating Station. This includes the development of material and the teaching of courses in both nuclear and non-nuclear aspects of the program.. He has recently been appointed as Chair of the Department of Chemical Engineering.