GENERAL MODELS OF DYNAMICAL SYSTEMS

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Contents

Summary

This article outlines certain important properties of dynamic systems, which happen to be the basis of their classification and characterization. The distinction between dynamic and static or steady-state behavior is explained. Properties such as time variance and time invariance, linearity and non-linearity, lumped and distributed parameters, continuous and discrete-time action, stability and instability etc. are discussed.

1. Mathematical Models

The mathematical equations used to describe the behavior of a system, based either on physical considerations or on measured phenomena, represent the mathematical model of the system. Mathematical models of dynamical systems are usually in the form of differential or difference equations together with algebraic or logical equations. Differential equations describe the system behavior in continuous time, whereas difference equations model dynamical systems in discrete time. The particular form depends on the actual nature of the system. Some important kinds of systems are schematically shown in Figure 1. There exist general approaches to modeling dynamical systems (see *Elements of Control Systems*)

Usually, mathematical models describe the input-output relations, which are useful in the analysis or synthesis of a control system as well as in the simulation studies. The simulation of a system may be made in analog, digital or hybrid computers to enable us to understand the performance of the system under different situations, while runs on actual systems may either be not feasible or may happen to be very expensive or risky to perform. System modeling, simulation and identification methods are useful in developing the knowledge that is prerequisite to control (see *Mathematical Models*).

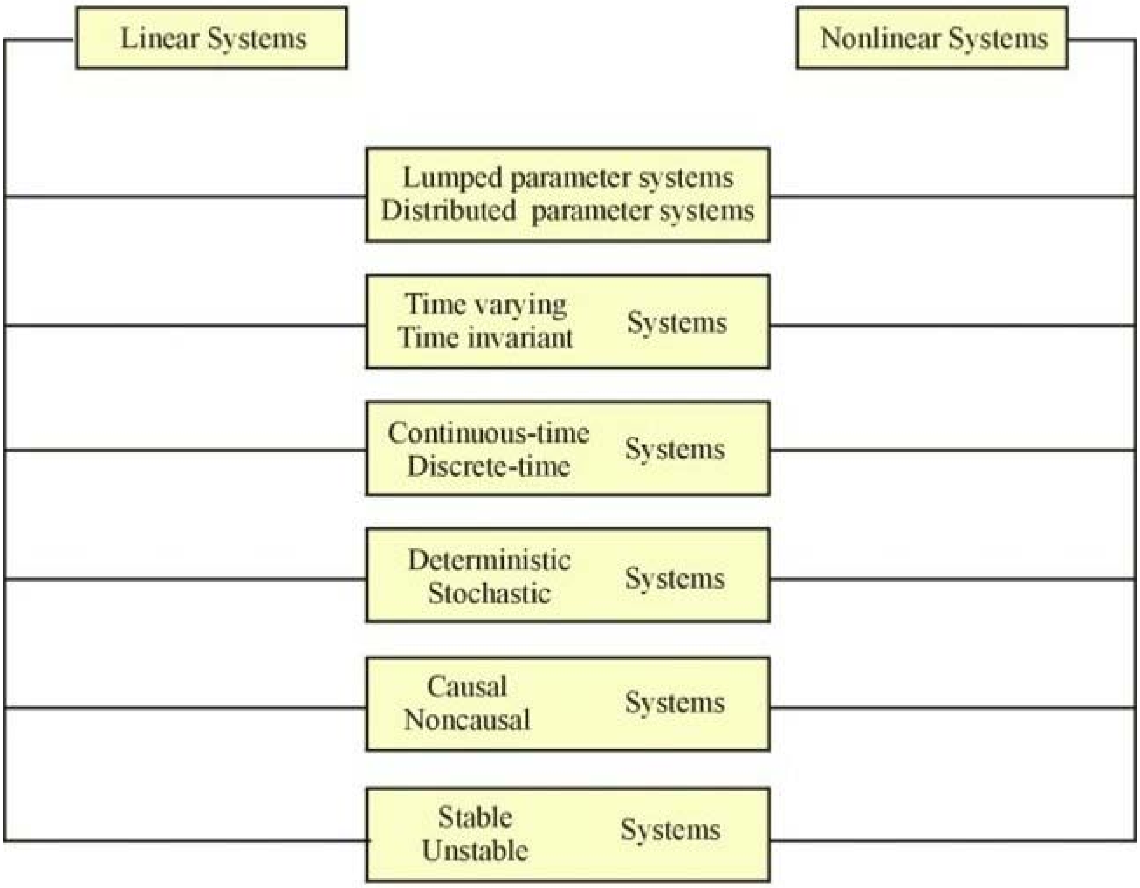

Figure 1. Several aspects of description of the properties of control systems

2. Dynamic and Static Behavior of Systems

The dynamic behavior or the time behavior of a system is exhibited by the system output $x_o(t)$ for a specified input function $x_i(t)$. This relationship between the input and output variables is generally expressed in terms of an operator T, i.e., for every real $x_i(t)$, there exists a real $x_o(t)$ such that

$$x_o(t) = T[x_i(t)]. \tag{1}$$

Consider the response $x_o(t)$ of a system for a 'step input', an input that is switched from zero to a unit value at t=0, as shown in Figure 2. Here $x_o(t)$ describes the time variation from the initial state at $t \leq 0$ to a steady final state $x_o(\infty)$. Varying the input step height

$x_{i,s}$ = constant, and noting the corresponding $x_{o,s} = x_o(\infty)$, the static characteristic $x_{o,s} = f(x_{i,s})$ that describes the steady state behavior of the system in a given region of operation, is obtained as shown in Figure 3.

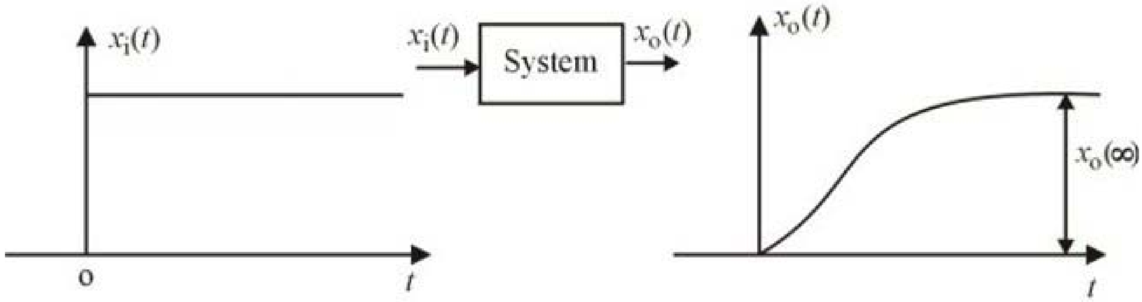

Figure 2. Dynamic behavior of a system

Figure 3. System behavior: (a) dynamic (b) static or steady-state

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Biographical Sketch

Ganti Prasada Rao was born in Seethanagaram, Andhra Pradesh, India, on August 25, 1942. He studied at the College of Engineering, Kakinada and received the B.E. degree in Electrical Engineering from Andhra University, Waltair, India in 1963, with first class and high honours. He received the M.Tech. (Control Systems Engineering) and Ph.D. degrees in Electrical Engineering in 1965 and 1970 respectively, both from the Indian Institute of Technology (IIT), Kharagpur, India. From July 1969 to October 1971, he was with the Department of Electrical Engineering, PSG College of Technology, Coimbatore, India as an Assistant Professor. In October 1971, he joined the Department of Electrical Engineering, IIT Kharagpur as an Assistant Professor and was a Professor there from May 1978 to June 1997. From May 1978 to August 1980, he was the Chairman of the Curriculum Development Cell

(Electrical Engineering) established by the Government of India at IIT Kharagpur. From October 1975 to July 1976, he was with the Control Systems Centre, University of Manchester Institute of Science and Technology (UMIST), Manchester, England, as a Commonwealth Postdoctoral Research Fellow. During October 1981- November 1983, May-June 1985 and May-June 1991, he visited the Lehrstuhl fuer Elektrische Steuerung und Regelung, Ruhr-Universitaet Bochum, Germany as a Research Fellow of the Alexander von Humboldt Foundation. Since June 1992 he is on a visit to Abu Dhabi as Scientific Advisor to the Directorate of Power and Desalination Plants, Water and Electricity Department, Government of Abu Dhabi and the International Foundation for Water Science and Technology where he worked in the field of desalination plant control. He is presently a member of the UNESCO-EOLSS Joint Committee.

He has authored/coauthored four books: *Piecewise Constant Orthogonal Functions And Their Applications to Systems and Control, Identification of Continuous Dynamical Systems- The Poisson Moment Functional (PMF) Approach* (with D.C.Saha) , *General Hybrid Orthogonal Functions and their Applications in Systems and Control* (with A. Patra) all the three published by Springer in 1983, 1983 and 1996 respectively*, and Identification of Continuous Systems* (with H.Unbehauen) Published by North Holland in 1987. He is Co-Editor (with N.K.Sinha) of *Identification of Continuous Systems - Methodology and Computer Implementation,* Kluwer, 1991. He has co-authored (with A.Patra): *General Hybrid Orthogonal Functions and Their Applications in Systems and Control,* LNCIS-213, Springer, 1996. He has authored/coauthored over 150 research papers. He is on the Editorial Boards of *International Journal of Modeling and Simulation, Control Theory and Advanced Technology (C-TAT), Systems Science (Poland), Systems Analysis Modeling and Simulation* (SAMS) and *The Students' Journal of IETE(India).* He was Guest Editor of two Special Issues: one of *C-TAT on Identification and Adaptive Control - Continuous Time Approaches,* Vol.9, N0.1, March 1993, and *The Students' Journal of IETE on Control*, Vols. I&II, 1992-93. He is on the Honorary Editorial Advisory Board of *The American Biographical Research Institute*. He organized several invited sessions in IFAC Symposia on Identification and System Parameter Estimation, 1988, 1991, 1994 and World Congress 1993. He was a member of the IFAC Technical Committee on Modeling, Identification and Signal Processing in 1996. He was Chairman of the Technical Committee of the 1989 National Systems Conference in India. He is co-editor (with A. Sydow) of the book series "*Numerical Insights Series*" published by Gordon and Breach. He is a member of the Advisory Board of the Internal Study Group on Water and Energy Systems (ISGWES). Over the last several years, he has devoted himself to the development, from concept to completion, of the *Encyclopedia of Desalination and Water Resources* (DESWARE-online) and *Encyclopedia of Life support Systems* (EOLSS), two major publications of EOLSS Publishers, Oxford, UK.

He has received several academic awards including the IIT Kharagpur Silver Jubilee Research Award 1985, The Systems Society of India Award 1989, International Desalination Association Best Paper Award 1995 and Honorary Professorship of the East China University of Science and Technology, Shanghai. The International Foundation for Water Science and Technology has established the 'Systems and Information Laboratory' in the Electrical Engineering Department at the Indian Institute of Technology, Kharagpur, in his honor. He is listed in several biographic publications. Professor Rao is a Life Fellow of The Institution of Engineers (India), Fellow of The Institution of Electronics and Telecommunication Engineers (India), Fellow of IEEE (USA) and a Fellow of the Indian National Academy of Engineering.